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DS-003-001205

Seat No. ________

B. Sc. (Sem. II) (CBCS) Examination

April / May – 2015

M-201 : Mathematics

(Geometry, Trigonometry & Matrix Algebra)

Faculty Code : 003
Subject Code : 001205

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

SECTION - A

1 Choose correct answer of the following M.C.Q's : **20**

(1) Equation of right circular cylinder whose axis is parallel to y-axis and radius r is ______

(A) $x^2 + z^2 = r^2$ (B) $x^2 + z^2 = y^2$

(C) $x^2 - y^2 + z^2 = r^2$ (D) $x^2 - y^2 = z^2$

(2) Radius of a right circular cylinder of which guiding curve is $x^2 + y^2 + z^2 = 4;\ \ x + y + z = 3$ is ______

(A) 1 (B) 4

(C) 5 (D) 3

(3) For any square matrix $A = \left[a_{ij}\right],\ a_{ij} = 0$ when $i \neq j$ is called -

(A) periodic matrix (B) diagonal matrix

(C) idempotent matrix (D) unitary matrix

(4) If matrices $A_{m \times n}$ and $B_{n \times m}$ then

(A) only AB defined (B) AB and BA both defined

(C) only BA defined (D) AB and BA not defined

(5) Each element of leading diagonal of skew-symmetric matrix is -

(A) 1 (B) 2

(C) 0 (D) 3

(6) Any matrix A is said to be non-singular iff

(A) $|A| = 0\,1$ (B) $A^T = A$

(C) $|A| \neq 0$ (D) $AA^T = I$

(7) For $n \times n$ matrices A and B, which of the following is true ?

(A) $A \cdot (adj\ A) = |A| \cdot I$ (B) $adj(AB) = adj\ B \cdot adj\ A$

(C) $(adj\ A)^T = adj\ A^T$ (D) All

(8) System of equation $AX = B$ is consistent iff

(A) Matrices A and B are of the same rank

(B) A is singular matrix

(C) A and $[A; B]$ do not have the same rank

(D) A and $[A; B]$ are of the same rank.

(9) The sequence $\{S_n\} = \{(-1)^n\}$ is

(A) oscillatory (B) convergent

(C) divergent (D) none of these

(10) Every convergent sequence

(A) is bounded (B) may not be bounded

(C) is Null sequence (D) must be increasing

(11) If $a = \text{cis}(2\alpha)$, $b = \text{cis}(2\beta)$, $c = \text{cis}(2\gamma)$, $d = \text{cis}(2\delta)$ then

$$\sqrt{a\,b\,c\,d} + \frac{1}{\sqrt{a\,b\,c\,d}} = \ldots\ldots\ldots\ldots$$

(A) $2\cos(\alpha + \beta + \gamma + \delta)$

(B) $\cos(2\alpha + 2\beta + 2\gamma + 2\delta)$

(C) $2\sin(2\alpha + 2\beta + 2\gamma + 2\delta)$

(D) $\text{cis}(2\alpha + 2\beta + 2\gamma + 2\delta)$

(12) $\sin x = \ldots\ldots\ldots\ldots$

(A) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots\ldots\ldots$

(B) $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots\ldots\ldots$

(C) $1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \ldots\ldots\ldots$

(D) $x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots\ldots\ldots$

(13) $(32)^{1/5} = \ldots\ldots\ldots\ldots$

(A) $2\left[\cos\left(\dfrac{2n\pi}{5}\right) - i\sin\left(\dfrac{2n\pi}{5}\right)\right]$

(B) $2\left[\cos\left(\dfrac{5n\pi}{2}\right) - i\sin\left(\dfrac{5n\pi}{2}\right)\right]$

(C) $2\left[\cos\left(\dfrac{2n\pi}{5}\right) + i\sin\left(\dfrac{2n\pi}{5}\right)\right]$

(D) $2\left[\cos\left(\dfrac{5n\pi}{2}\right) + i\sin\left(\dfrac{5n\pi}{2}\right)\right]$

(14) $\cos 7\theta = \ldots\ldots\ldots\ldots$

(A) $\cos^7\theta + 21\cos^5\theta\sin^2\theta + 5\cos^3\theta\sin^4\theta + 7\cos\theta\sin\theta$

(B) $\cos^7\theta + 18\cos^5\theta\sin^2\theta + 15\cos^3\theta\sin^4\theta + 7\cos\theta\sin^6\theta$

(C) $\cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta$

(D) $\cos^7\theta - 21\cos^5\theta\sin^2\theta - 25\cos^3\theta\sin^4\theta - 9\cos\theta\sin^6\theta$

(15) $\tan x = ...$

(A) $x - \frac{x^3}{3!} + \frac{2}{5!}x^5 - ....$

(B) $x + \frac{x^3}{3!} + \frac{2}{5!}x^5 + ...$

(C) $x + \frac{x^3}{3} + \frac{2}{15}x^5 + ...$

(D) $1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \frac{x^6}{6!} - ....$

(16) $(\cos\theta - i\sin\theta)^{-10} = ...$

(A) $\text{cis}\,10\,\theta$ (B) $\cos 10\,\theta + i\sin 10\,\theta$

(C) Both (A) and (B) (D) $\cos 10\,\theta - i\sin 10\,\theta$

(17) $\sin i\theta = ...$

(A) $\sinh\theta$ (B) $i\sinh\theta$

(C) $\sin\theta$ (D) $-\sin\theta$

(18) The real part of $\log(5+12i)$ is ....

(A) $\log 12$ (B) $\log 13$

(C) $\log 5$ (D) $\log 15$

(19) $\log(-5) = ...$

(A) $\log 5$ (B) $\log 5 - \pi i$

(C) $\frac{1}{2}\log 5$ (D) $\frac{1}{5}\log 2$

(20) $(\text{cis}\,2\theta)^4 \cdot (\text{cis}\,4\theta)^3 = ...$

(A) $\text{cis}\,10\,\theta$ (B) $\text{cis}\,20\,\theta$

(C) $\text{cis}\,12\,\theta$ (D) $\text{cis}\,8\,\theta$

SECTION - B

2 (a) Attempt any **three** : **6**

(1) Define : Right circular cylinder, Eigen values.

(2) Find equation of right circular cylinder whose radius is 2 and axis is $\frac{x-1}{2}=\frac{y}{3}=z-3$.

(3) If Matrix A is idempotent then show that matrix $I-A$ is also idempotent.

(4) Check whether matrix $A=\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ is involuntary or not.

(5) Determine whether the following sequences are convergent or not.

(i) $\left\{1+\frac{(-1)^n}{n}\right\}$

(ii) $\{\sin n\pi\}$

(6) Define : Convergent sequence, Cauchy sequence.

(b) Attempt any three : **9**

(1) Find equation of cylinder whose generator is parallel to $\frac{x}{\ell}=\frac{y}{m}=\frac{z}{n}$ and enveloping curve is $x^2+y^2+z^2=a^2$.

(2) Find equation of cylinder whose generator is parallel to the straight line $x=\frac{-y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+2y^2=1; z=3$.

(3) If A, B, C are matrices of the type $m \times n,\ n \times p,\ p \times q$ respectively then prove that $(AB)C = A(BC)$.

(4) Find period of matrix $A = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(5) Prove that Every convergent sequence is bounded.

(6) Show that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2S_n}$ is convergent and find its limit.

(c) Attempt any **two** : **10**

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

(2) Find the rank of matrix $A = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$.

(3) Solve following linear equations :

$x + y + z = 9$

$2x + 5y + 7z = 52$

$2x + y - z = 0$

(4) If $\lim\limits_{n \to \infty} a_n = \ell$ then show that

$$\lim_{n \to \infty} \left(\frac{a_1 + a_2 + \ldots\ldots + a_n}{n} \right) = \ell.$$

(5) Find : $\lim\limits_{n \to \infty} \frac{1}{n} \left(1 + 2^{1/2} + 3^{1/3} + \ldots\ldots\ldots + n^{1/n} \right)$

3 (a) Attempt any **three** : **6**

(1) Simplify : $\dfrac{\left(\cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}\right)^5}{\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)^{1/2}}$

(2) Evaluate : $\left(\sqrt{3}+i\right)^7 + \left(\sqrt{3}-i\right)^7$

(3) Prove that

$$\sin 8\theta \cdot \cos ec\theta = 128\cos^7\theta - 192\cos^5\theta + 80\cos^3\theta - 8\cos\theta$$

(4) Solve : $7\sinh x + 20\cosh x = 24$.

(5) Find real and imaginary parts of i^i.

(6) Prove that $\tanh^{-1} x = \sinh^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$.

(b) Attempt any **three** : **9**

(1) Prove that $(1+i)^n + (1-i)^n = 2^{\left(\frac{n}{2}+1\right)} \cdot \cos\dfrac{n\pi}{4}$

(2) Expand $\cos\alpha$ interms of α.

(3) Prove that the roots of the equation

$8x^3 - 4x^2 - 4x + 1 = 0$ are $\cos\dfrac{\pi}{7}$, $\cos\dfrac{3\pi}{7}$ and $\cos\dfrac{5\pi}{7}$.

(4) Prove that $\sin(x+y) = \sin x\cos y + \cos x\sin y$.

(5) If $5\cosh z - 13 = 0$ then find value of z.

(6) Separate $\sin^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts.

(c) Attempt any two : **10**

(1) $i^{i^{i \cdots}} = a + ib$ then by using principal value. Prove that $a^2 + b^2 = e^{-\pi b}$, $b = a \tan \frac{\pi a}{2}$.

(2) $A + iB = K \tan(x + iy)$ then prove that

$$\tan 2x = \frac{2KA}{K^2 - A^2 - B^2}$$

(3) Prove that

$$\left[\frac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta}\right]^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$$

(4) If $\tan\theta = \frac{1}{2}$ then find $\tan 6\theta$.

(5) If $\sin 6\theta = a\cos^5\theta\sin\theta + b\cos^3\theta\sin^3\theta + c\cos\theta\sin^5\theta$ then find value of a, b and c.
